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# POSSIBLE DETERMINATION OF AN UNSTEADY HEAT FLUX BY USING THE FINAL STATE OF TEMPERATURE INDICATORS LOCATED ALONG THE SAMPLE DEPTH*

A. K. Alekseev

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*The problem of determining the heat flux density as a function of time by using the final value of the coordinates of the regions of change in color of temperature indicators located along the sample depth is considered.*

For some practical problems, when for any reasons it is impossible or undesirable to measure the heat flux density or the temperature by direct methods, it is of interest to determine the heating history of a sample by using its state after completion of heating without using external measuring facilities.

By heating history we mean the heat flux density over time $q_w(t)$ acting on a sample or the temperature on its surface $T_w(t)$.

In many practical problems, methods of determining the temperature of structural elements by using special substances that irreversibly fix changes in their structure under heating are employed. Temperature indicators, irradiated crystals, and some other substances are used for determining the maximum temperature. In the case where the time function of the heat flux is known beforehand, the acting heat flux can be recovered by using data on the maximum temperature. Determining a heat flux whose dependence on time is unknown by using irreversible material changes is also of practical interest.

In [1], the problem of determining $q_w(t)$ by using the extent of the thermodestruction of a material was considered. It leads to problems in identifying a thermodestructible material model and determining the parameters of this model and to an inverse heat transfer problem for the thermodestructible material and requires complex experiments on thermogravimetric analysis.

In the present article, another procedure - use of strips of temperature indicators of fusion [2] (or fusible inserts) located along the material depth (in the heat propagation direction) (see Fig. 1) - is considered for heating process detection. When temperature indicators are used in the usual manner, they are applied to the outer (heated) or to the inner (heat-insulated) surface. In this case, a maximum temperature is recorded on these surfaces.

In the proposed investigation, the coordinates of the region of transition (change in color) of the temperature indicators (they may be interpreted as the maximum-temperature distribution along the material

* In publishing this article the Editorial Board considers it necessary to present a referee's opinion: "The author proposes an original statement of the inverse heat conduction problem on determining a time-dependent heat flux when the final values of the coordinates of the regions of change in color of temperature indicators located along the material depth - in the heat propagation direction - are used as initial experimental data. The possibility that this problem has more than one solution is discussed. This is the main difficulty in such an approach. It is suggested that this uncertainty by circumvented by using a priori information on the solution class. "Numerical results show that this appreach is possible in principle, although further studies are needed. "Taking into account the new nontraditional statement of the problem and interesting initial results relating to it, I think that this article can be published in the Journal of Engineering Physics despite the fact that it is definitely vulnerable to criticism." O. M. Alifanov.

Scientific Production Association "Energy," Moscow, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 65, No. 2, pp. 243-248, August, 1993. Original article submitted July 30, 1992.

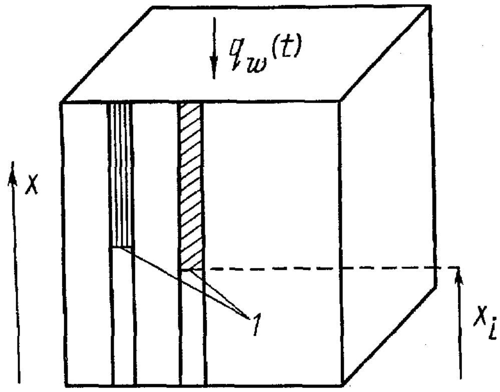



Fig. 1. Diagram of the layout of the temperature indicators in a sampole and measurements of the coordinates of regions of change in color: 1) regions of change in color of the temperature indicators.

depth) are recorded. We consider a one-dimensional plate that is heated by a heat flux $q_w(t)$ and heat-insulated from the inside. Temperature indicator strips with different transition temperatures are arranged along the material depth (normal to the isotherms). Technically, this can be achieved, e.g., as follows: having cut out an insert from a sample, apply temperature indicator strips to its side surface along the depth and then paste it in its old place.

The arrangement of the temperature indicator strips along the material depth makes it possible to record the transition front coordinates of the i-th temperature indicator $X(T_i)$ after a sample has cooled off.

The information on heating ($q_w(t)$) is converted to a temperature field $T(t, x)$ by using the heat conduction equation and is recorded by the temperature indicators in the form $X(T_i)$.

We formulate the problem of recovering the heat flux density $q_w(t)$ along the transition front coordinates of the temperature indicators $X(T_i)$.

Let the temperature field be recorded during heating as follows: the sign of transition of the i-th temperature indicator $IND_i(x) = \int_0^{tk} \delta(T(t, x) - T_i)dt$ equals zero before a transition and unity after it. The transition front coordinates for a set of N temperature indicators $X(T_i)$ ($i = 1, ..., N$) will be considered as a maximum-temperature distribution along the coordinate for the time of the process in the form $T_{max}(x) = f(x)$.

We consider the simplest model for heat transfer (without allowing for the effect of the heat of a phase transition or the thermophysical properties of the temperature indicator on the sample temperature field). We write a one-dimensional heat conduction equation with appropriate initial and boundary conditions:

$$C\rho \frac{\partial T(t, x)}{\partial t} = \lambda \frac{\partial^2 T(t, x)}{\partial^2 x}; \quad (1)$$

$$\lambda \frac{\partial T}{\partial x}\bigg|_{x=L} = q_w(t); \quad \frac{\partial T}{\partial x}\bigg|_{x=0} = 0; \quad (2)$$

$$x \in (0, L); \quad t \in (0, tk); \quad T(0, x) = T_0(x). \quad (3)$$

Experiment gives us the field of the signs of transition (at the moment of final observation):

$$IND_i(x) = \int_0^{tk} \delta(T(t, x) - T_i)\, dt, \quad i = 1, \dots, N,$$

or the maximum-temperature distribution:

$$\max_{t \in (0, tk)} (T(t, x)) = T_{max}(x) = f(x). \quad (4)$$

We need to have a heat flux $q_w(t)$ such that $IND_i(x)$ or $f(x)$ can be recovered.

In essence, we have here a degenerate problem of heat transfer in a medium with several phase transition fronts when the phase transition is fixed. This problem relates to the class of inverse heat transfer problems [3]

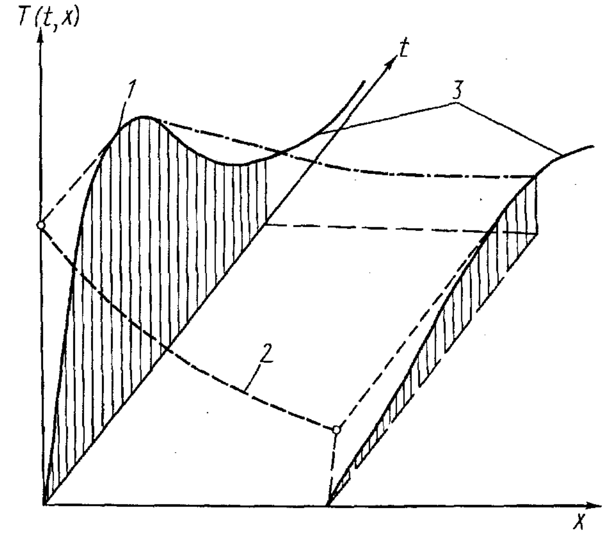



Fig. 2. Determination of the surface temperature $T_w(t)$ by using the maximum-temperature distribution $T_{max}(x)$: 1) $T_w(t)$; 2) $T_{max}(x)$; 3) "shadow" region.

and is incorrectly stated. As compared to the majority of inverse problems, the question of uniqueness is of special importance for this problem.

It is easy to see examples of nonuniqueness: in the case of two heating pulses separated in time by a cooling process, one of them may partially or completely screen the other, as a result of which many functions $q_w(t)$ can correspond to one final state. Here, the nonuniqueness is related to the projection nature of fixing the process.

We consider the problem of how the unknown signal (the heat flux $q_w(t)$) is transformed and coded by the heat conduction equation and the phase transition.

Let we have a time-variable heat flux $q_w(t)$ and a maximum-temperature profile along the sample depth.

We consider a finite-difference approximation $q_w(t_j) = q_j$, $T(t_j) = T_j$ $(j = 1, ..., N)$. The heat flux $q_j$ is converted to the temperature $T_j(x_i)$ on each layer along the coordinate $x_i$:

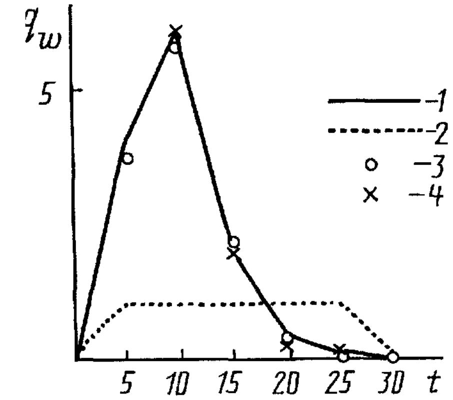
$$T_j(x_i) = \Phi_i q_j.$$

Here $\Phi_i$ is the heat transfer operator. From Duhamel's principle, $\Phi_i$ is of the form (for a uniform time step):

$$\Phi_i = \begin{pmatrix} a_i & 0 & 0 & 0 \\ b_i & a_i & 0 & 0 \\ c_i & b_i & a_i & 0 \\ d_i & c_i & b_i & a_i \end{pmatrix}$$

The coefficients a, b, c, ... are nonlinear functions of X and t. Then a maximal element $T_{max}$ is chosen from a converted signal $q_w(t)$, is stored in material, and serves as initial data in problem (1)-(4). This choice already depends on the form of the solution. And depending on this choice, the solution to (1)-(4) may be unique or not. We obtain a system of equations with linearly independent coefficients, which may, however, be underdetermined because of zero columns. For example, for a heat effect having two maxima, the first of which is much greater than the second and which are separated by a time interval sufficient to equalize the temperature, a degenerate matrix of the type

$$A = \begin{pmatrix} a_1 & 0 & 0 & 0 & . & 0 & 0 \\ b_2 & a_2 & 0 & 0 & . & 0 & 0 \\ c_3 & b_3 & a_3 & 0 & . & 0 & 0 \\ d_4 & d_4 & b_4 & a_4 & . & 0 & 0 \\ . & . & . & . & . & . & . \\ d_N & c_N & b_N & a_N & . & 0 & 0 \end{pmatrix}$$



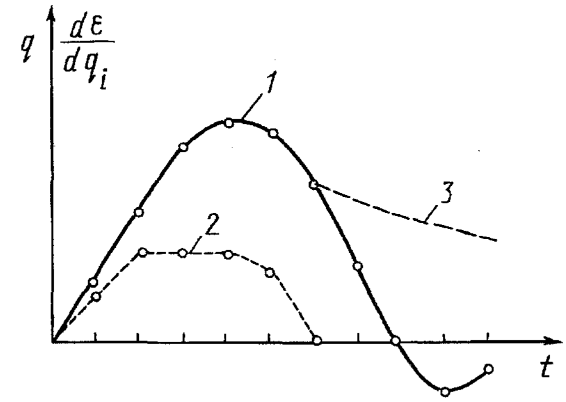




Fig. 3. Recovery of the heat flux density along the coordinates of the regions of change in color of the temperature indicators: 1) exact value; 2) initial approximation; 3) calculation results with exact initial data; 4) calculation results with perturbed initial data (1% error). $q_w$, kW/m$^2$; t, sec.

Fig. 4. Determination of the maximum-estimate domain by using a numerical solution: 1) exact solution; 2) magnitude of the sensitivity of the discrepancy to the solution components; 3) maximum estimate in the shadow domain.

with zero columns on the right will be obtained. We have lost some part of the unknown signal. A priori information is needed to recover it. We have here the case of nonuniqueness, whose physical meaning consists in a loss of sensitivity to secondary weak heating.

The question of uniqueness should be solved by proceeding from the available a priori information on the class to which the desired solution (process duration, signal behavior on different sections, etc.) belongs.

In the case of nonuniqueness, part of the information is lost. This mainly relates to negative heat fluxes and fluxes after a heating maximum has passed (see Fig. 2).

It should be noted that the difference in the mechanisms of fixing irreversible processes (phase transition and kinetic process)

$$\mathrm{IND}_i(x) = \int_0^{t_k} \delta(T(t, x) - T_i)\, dt, \quad i = 1, \ldots, N;$$

$$C_i(x) = \int_0^{t_k} F_i(T(t, x))\, dt, \quad i = 1, \ldots, N,$$

between the present article and [4] gives substantially different results for uniqueness. The kinetic process allows a larger volume of information to be stored than the phase transition, and it allows the above-mentioned degeneration to be avoided.

The stated problem was analyzed by using numerical experiments. A heat flux density vector $q_j$ ($j = 1, \ldots, N$) parametrizing some function ($q_j = q(t_j)$) as a vector that minimizes the discrepancy $\varepsilon = \sum_{i=1}^{N} (f(x_i) - f_{cal}(x_i))^2$ between the calculated and experimental values of a maximum temperature was sought. The conjugated gradient method was used to find a minimum. Derivatives of the discrepancy functional were obtained by the difference approximation with respect to the parameters $q_j$. Computations were done on a computer (BESM-6) and took 1-5 h. Problem (1)-(3) was solved by the integrointerpolation method. Regularizing corrections (of zero and first orders) were introduced into the discrepancy.

The numerical experiments supported the possibility of successfully solving the proposed system of equations (see Fig. 3) as applied to heat fluxes of rather simple form. In the case of degeneration, the part of the solution that is in the "shadow" domain and does not affect Tmax(x) was found by using the value of the discrepancy gradient components (equal to zero).

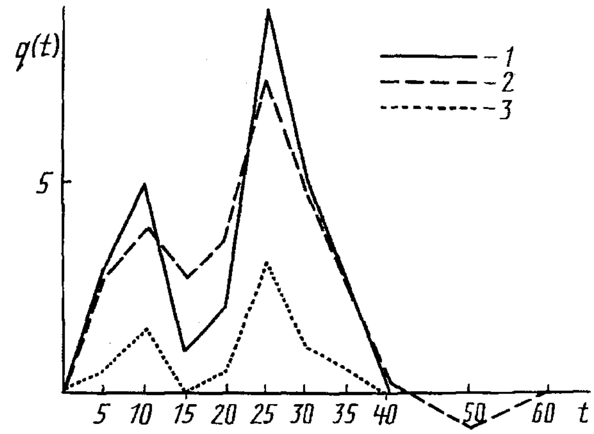



Fig. 5. Recovery of the heat flux density along the coordinates of regions of change color of the temperature indicators: 1) exact value; 2) calculation results; 3) initial approximation. q(t), $kW/m^2$.

Thus, having obtained some solution we can determine whether there is a the "shadow" domain, in which a change in $q_w(t)$ does not affect $T_{max}(x)$, in this solution by using the discrepancy gradient. If it exists, then the boundary of this domain from above can be found by using $q_w(t)$. This will be an estimate of the maximum heat flux in this domain (see Fig. 4).

The discrepancy has a ravine structure (the discrepancy changes much more weakly along the direction that preserves the total heat flux value).

The solution of the above-stated problems will be affected significantly by the transition temperature error of the temperature indicators of fusion ($\pm 1.0\%$) at low temperatures (up to 500°C) [2], by heat transfer in the region of the temperature indicators and the difference of their thermophysics from those of the base material, and by the temperature gradient across the test layer (the accuracy in determining the transition point coordinate).

The effect of the errors in the initial data on the accuracy of the solution was examined by numerical experiments (the exact data were perturbed by a random error amounting to $\pm 1.0\%$). For rather simple dependences q(t) (with one maximum) good accuracy of recovering of the heat flux ($q_w(t)$) was obtained (see Fig. 3). It should be noted that on increase in the error in the initial data markedly reduces the rate of convergence of the gradient methods. For more complex dependences $q_w(t)$ (for example, multiextreme ones) the error in recovering the heat flux $q_w(t)$ is much higher and attains tens of percents (Fig. 5).

The above results indicate the possibility of measuring (or maximum estimation of) the heat flux density over time with a solution to the inverse heat transfer problem by using measurements of the transition region coordinates of the temperature indicators, when a priori information on the nature of the thermal effect is available.

## NOTATION

t, time; x, coordinate; T(t, x), temperature; q(t), heat flux density; $\lambda$, thermal conductivity; $\rho$, density; C, heat capacity; $T_i$, transition temperature of the i-th temperature indicator; $\delta$, Dirac delta function. Subscripts: max, maximum value; w, heated boundary; i, number of a temperature indicator; cal, calculated value.

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